Ramon Llull's contributions to computer science

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Abstract. The task that preoccupied Ramon Llull throughout his life (1232–1316), and which is now referred to as the Great Art *Ars magna*, was a philosophical system proposing a system for raising, in an objective manner, questions about God and the world. In it he introduced a new system of logic. In the course of defining his *Art* he made significant contributions to the development of the discipline that is now known as computer science. Some of these contributions are also relevant to mathematics. The latter have become well known, principally because his ideas were taken up by Leibniz (1646-1716) and his followers. We discuss Llull's remarkably innovative contributions.

1 Introduction

In 1958 Martin Gardner [13] was the first modern to draw attention to Llull's contributions to logic machines in his book *Logic machines and diagrams* [13]. Llull's contributions to what has become mathematical logic and computer science have been noted by various authors over the years, but their assessments have oscillated between brief dismissal and adulation. Thus Prantl ends a long exposition with a sharp dismissal of Llull: *Dass die ganze Kunst des Lullus schlechthin werthlos ist, bedarf nun wohl keines besonderen Nachweises mehr.*[1] On the other hand Sowa [34], pp.5–6, graciously acknowledges one of Llull's contributions to computer science in that Llull's rotating discs generated large numbers of combinations which could then be tested. Others, such as Bonner [2], pp.65 ff. have regarded Llull as giving rise to the modern theory of graphs but this seems hard to substantiate. Certainly Llull drew interconnections between concepts, but he did not undertake any analysis of the graphs themselves. Llull has been referred to as the "first computer scientist" in particular by Catalans, but there is also an extensive study by Künzel [17]. Ton Sales [32] describes Llull as "One of Us", where "Us" means Computer Scientists. Likewise Anthony

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[1] [28], vol.3, pp.145–177, at p.177. "That the whole art of Lullus is worthless now needs no more specific evidence."

Bonner [3] asks "What was Llull up to?" and presents a nice picture from the point of view of a modern computer scientist. However, it is always dangerous to take the concepts of the present day back into medieval times. Indeed, the study of the history of science over a long time (meaning both the student's and historical time) shows how slowly seemingly simple and seminal ideas develop. The classical example of misjudgment is the attribution to Roger Bacon (1214?-1294) of the invention of the "Scientific Method", since this method can be seen developing over at least two or three centuries and some would claim that traces can even be seen in Aristotle. Llull himself would never, indeed could never, have described himself as a "computer scientist". The very phrase, if not the words, was not in circulation, and neither were the appropriate concepts current.

Nevertheless some ideas of Llull were explicitly used by Leibniz (1646–1716) in [20] (where Llull is mentioned by name several times) and before him by Kircher [16], and led directly into the development of computing machines through Llull's development of a formal language and the mechanics of generating combinations. But Llull contributed more than that. He also contributed to the logical notion of a formal language, and to the mathematical notions of variable and substitution (for a variable).

In this article I discuss Llull's contributions and I do this from an abstract point of view.[2] From today's point of view these are to the following areas:

1. the idea of a formal language,
2. the idea of a logical rule,
3. the computation of combinations,
4. the use of binary and ternary relations,
5. the use of symbols for variables,
6. the idea of substitution for a variable,
7. the use of a machine for logic.

Finally I consider the rôle of Llull in relation to the modern disciplines of computer science and computer engineering.

The first question then is: what was Llull's aim in developing his *Great Art*? The answer is simple: the conversion of non-Christians, but meaning specifically those who were of the religions of the book, i.e. Jews and Moslems. This aim was to be achieved by starting in an objective world and then asking questions. Llull believed that the solution of these questions would then convince the hearers of the truth of Christianity. Therefore he spent a very large part of his life designing the appropriate (perfect, to use Eco's word in [11]) language.[3] In this paper I shall only discuss the final version of Llull's magnum opus.

[2] When I refer to "Llull's methodology" I mean the way in which he obtained, or thought of, the devices he employed, rather than how to use Llull's system. For the latter there is an excellent guide in [4].
[3] Of course Eco was not the first or only person to use this adjective. See, e.g. [31].

2 Llull's motivation

In looking at the contributions that Llull made and which led into computer science it is important to realize three things. First, that the construction of Llull's new logic took place throughout Llull's life. Secondly, that Llull's aim in developing his system was not to develop logic but to provide a means for the conversion of monotheistic non-Christians, specifically Jews and Moslems.[4] Thirdly, the mechanical systems of Llull did not provide answers to the questions that his machine generated. Rather, having posed the question, Llull believed that it would then be possible to use the usual techniques of argument with the result that the Christian's argument would lead to the conversion of the non-Christian. As Colomer [8], p.118, writes: *El mecanisme lògic de l'Art està al servei d'una finalitat religiosa.* Thus Llull would begin in an objective world and then ask questions. (See [24], p.XVII, and the *Prologue*, p. 3, of the *Ars Brevis.*) The objectivity would avoid the Christian taking a superior position and would promote engagement with the interlocutor. The solution of the questions would then convince the hearer of the truth of Christianity. However, we should note that Llull's idea of logic is not purely abstract; it is abstracted from reality but also contains facts. As Rossi [31], p. 32, puts it: "While metaphysics considers entities external to the soul 'from the point of view of their being' and logic considers them according to the being which they have in the soul, the art – supreme among all the sciences – considers entities in both ways at once.[5]"

In order for his system to be effective it was necessary for it to be understood. This seems to have caused Llull considerable difficulty and is, presumably, the reason why he simplified his system over the years, and also why he learnt Arabic.

It seems appropriate to regard Llull's work as being logical in the sense used by Leibniz, as well as the contemporary sense (which is not too different). Llull himself says (*Ars Brevis*, see [24], chapter XII, Form 87):

> *Logica est ars, cum qua logicus invenit naturalem coniunctionem inter subiectum et praedicatum.*

In the sections that follow we first recall the modern approach and then view Llull's work from this perspective. We shall principally restrict ourselves to Llull's *Ars Brevis* [24] since the points that we which to make are all included there.

3 Formal languages

In modern formal logic (otherwise called "symbolic" or "mathematical") the basic expressions of the syntax of the language begin with primitive elements, from which are built "atoms" or "atomic formulae" by putting them together with the aid of connectives: & ("and"), $\rightarrow$ ("implies"), $\vee$ ("or"), $\neg$ ("not"), and

[4] This is eloquently expressed by the late Robert Pring-Mill in [29].
[5] Rossi has a note giving the reference in [22], vol. III, p.1.

so on. The expressions so obtained are called "well-formed formulae" or simply "formulae". Rules (see below, section 4.2) may then be used to deduce more formulae. The formulae have interpretations, i.e. a semantics. Thus $p \vee \neg p$, which is read "p or not p" is interpreted in the obvious way if we have a proposition interpreting p, for example: "It is raining or it is not the case that it is raining".[6]

Llull gave formal procedures for the language that he used. There were a finite number of primitives. Originally he started with many more but in the last analysis these were reduced to nine which form his *alphabet*: B, C, D, E, F, G, H, J, K.[7,8] These primitives will be interpreted in different ways. We discuss this in detail below in section 7 but for now it suffices to give an example: B is interpreted as goodness, difference, whether?, God, justice or avarice, C as size (or greatness), concordance, what?, angels, prudence, gluttony.

> *B significat bonitatem, differentiam, utrum, Deum, iustitiam et avaritiam.*
> *C significat magnitudinem, concordantiam, quid?, angelum, prudentiam et gulam.*

It should be noted (for Section 4.2 below) that each letter has an interrogative word included in the list of interpretations.

It is very interesting that Llull should have opted for a finite number of primitives since Llull's aim is to be able to produce all questions. Later, when we come to combinations, we shall see that he was certainly interested in producing large numbers of combinations, though he never refers to infinitely many being possible. Having determined the primitives he puts these together according to his four *figures*. One can justifiably say that he puts them together according to mechanical rules (see sections 4 and 8 below). These combinations can then be read to give the various possible questions.

3.1 Llull's treatment of language

First we should distinguish between the formulae, namely the combinations of letters, on the one hand, and the interpretation of these combinations on the other. One is tempted to read his formulae as simple combinations BC, EK, etc. but this has to be interpreted in the context of the particular figure being used. Secondly, each individual letter is interpreted not only as an abstract noun, e.g. *bonitas* but also as the corresponding adjective, here *bonum*. On occasion he goes further and makes verbs out of nouns. This is difficult to render in both Latin

[6] It should be noted that in modern logic the "or" is the "inclusive or".

[7] The letter A had already been used for the name of Llull's first figure. It is interesting that although Llull did not use suggestive notation, that is to say, dividing the alphabet into different parts for different uses, nevertheless he retained his letterings throughout his work. This perhaps explains why the figures themselves are labelled with letters quite remote from each other in the Roman alphabet.

[8] The relevant *Figures* referred to in this paper may be found at `http://lullianarts.net/Ars-Magna/ars-magna.htm`.

and English. Thus in chapter IX when dealing with the fourth subject, which is Man, we find:

> *Homo est compositus ex anima et corpore. Ratione cuius deductibilis est per principia et regulas duobus modis, videlicet modo spirituali et modo corporali. Et definitur sic: Homo est animal homificans.*[9]

Following from this second point, we note that Llull, explicitly, interchanges subject and predicate. This is completely outside the normal practice of logicians throughout history. Indeed, apart from Llull, this idea of treating subjects and predicates on the same level was not used until the twentieth century when Henkin (1921–2006) introduced it in his paper [15]. This is already clear in the case of the first figure which we discuss in the next section, and explicit in the treatment of the third figure in chapter VI.

> *... mutando subiectum in praedicatum.*[10]

Besides predicates, which traditionally since Aristotle, took only one argument (to use the modern terminology), Llull introduces binary and ternary relations. However, these are only in the context of his formulae, see Section 6 below.

4 Llull's figures

Llull introduces *figures*, (see `http://lullianarts.net/Ars-Magna/ars-magna.htm`) as he calls them, to yield combinations of letters. Figure A joins primitive elements by making one the subject and one the predicate. Thus *bonitas* is joined with *magna*, rather than with *magnitudo* to give *Bonitas est magna*.

It is clear from his figure A, and the accompanying text, that *bonum* is the neuter form of the adjective, not the abstract noun meaning "a good thing". Later we find him creating new verbs from nouns in the ninth part of the *Ars magna* (see below section 4.2).

Since there is no priority among the letters we can also join *magnitudo* with *bona* to get *Magnitudo est bona*. This also permits such conjunctions as *Avaritia est bona* which, since Llull says "avarice is not good but evil" (*Ars generalis ultima*, [23], chapter I, 2), shows that these are not true statements, but simply statements, or, as Llull generally uses them, questions.

Thus Llull has provided a method for producing such statements. In modern language this is: If x and y are primitive elements (letters of the Alphabet) then "x est y" is a statement (formula). This form is just like the modern form "x and y", or more symbolically "$x \,\&\, y$".

The second figure is more complicated to apply. It is denoted by T simply because it came after figure S in earlier versions of Llull's work and the letter

[9] "A man is composed of a soul and a body. For this reason he can be studied using the principles and rules in two ways: namely in a spiritual way and in a physical way. And he is defined thus: man is a man-making animal."

[10] ... exchanging subjects with predicates.

T comes immediately after S in the Roman alphabet. The inner triangles each contribute in two ways to the formation of statements. First each side of a triangle connects two letters together; secondly it brings an additional quality into the formula. For example, in the first triangle we have difference, concordance and contrariety. Any one of these will record a type of conjunction between a letter of the alphabet and another one. For example, "goodness is different from greatness". We could symbolise this in a modern guise by *Diff*(x, y). Here Llull is introducing binary relations as opposed to predicates (see also Section 6 below).

The third figure continues this process, adding further complexity. This is also where Llull's preoccupation with combinations emerges. The third figure exhibits all the combinations of two letters from the Alphabet, except that identical letter combinations, such as BB, are not permitted. Further Llull does not regard BC, for example, as different from CB and therefore the number of combinations is reduced to half. A modern calculation gives $9 \times 8 = 72$ combinations from which we remove half of these because the order of the *two* letters does not matter. The result is the 36 that Llull has. However, Llull's arrangement of the combinations in his step or triangular diagram suggests that his approach was different. The rows have $8, 7, 6, \ldots, 1$ entries and $8 + 7 + 6 + \ldots + 2 + 1 = 36$. Hidden under this is the fact that each letter of his Alphabet can also be interpreted according to the interpretation in either the first or second figures, as previously discussed.

In the fourth figure the process is continued. Here we genuinely have ternary relations (see also Section 6 below). He produces combinations of three letters, where there is a restriction on repeating a letter in the trio. He also has other rules that exclude certain other combinations so we do not get all the possible combinations of three letters.

The details of the interpretation are not spelt out in his treatment of the third figure. This is reserved until he has introduced his definitions and rules. Then he gives the full and explicit interpretation when he describes his *Table*.

4.1 Principles

The principles are what we would call *axioms*. For Llull these are not formulae but statements involving the terms of the alphabet. *Cum talibus enim condicionibus intellectus facit scientia. ...*"[11] The first and tenth are:

> *1. Goodness is that whereby good does good.*
> *10. Difference is that by reason of which goodness, etc. are distinct and clear notions.*

The principles appear to have been chosen in the light of his experience–perhaps in the same way that the laws of logic, even from the time of Aristotle, have been established.[12] That Llull, despite all his work, did not have a clear idea of

[11] [24], pp. 26–27. "For with such conditions the intellect acquires knowledge, ..."

[12] I am aware that the laws of logic are regarded as necessarily true, but modern logic has invented many different kind of logics, with different laws, that are clearly valid in appropriate domains of discourse. Therefore I would maintain that although the

the relevant status of his principles is evident from his statement at the end of part 3 in the *Ars generalis ultima*, [23]. Here he claims that his principles help to clarify what is going on and to provide guidance in the way that a statement may be resolved to be true or false.

> *Now there are those who dare to attack our principles with canine fangs and serpentine tongue, as they disparage and slander our definitions. However, the art has principles that mutually help each other, for instance when someone says: "If greatness is the being on account of which goodness is great, then all goodness must be equally great;" this can be refuted with the principles of majority, minority and contrariety which do not allow every kind of goodness to be equally great.*

Today we would say that these rules are rather meta-rules, governing the working of the logic (and generally accepted).

4.2 Rules

In a modern system of formal logic, having built the formulae we then have a set of rules by means of which we can deduce (or infer) more formulae. The aim in modern logic is to go from true formulae to true formulae. Llull does indeed have rules, but their purpose is to generate more statements (or questions). However, they do not have the same form as rules of modern logic. Rather they are more like the basic rules for building up formulae. Thus, in modern logic one says, 1. a basic (or atomic) formula is a (well-formed)[13] formula, 2. if A and B are formulae, then $A\&B$ is a formula. In Llull only one iteration of the second step is allowed. The rules are purely syntactic; the semantic considerations are left entirely to the user.

There are ten rules, one associated with each letter except there are two for K, and all using the question word associated with that letter.

As noted earlier, every combination of letters gives rise to many different statements (or questions), for example, BC using the first figure, and BTCD from the fourth figure. Indeed, this whole process gives rise to a very large number of statements (or questions). Not only do we have the various possibilities for each letter, we also have different ways of ordering the letters. Llull provides his *Table* of the allowed combinations.

5 Llull's *Table*

The *Table* lists all combinations of three letters from the fourth figure and each is put in a cell (*camera*). In addition the letter T is included but it has a very

laws of logic have withstood the test of time, nevertheless they are dependent on the kind of world in which we live, and the kind of creature that we are. Llull thought over these principles for a very long time and gives no direct justification for them while maintaining that they are essential. See also footnote 1, p. 309 of [1].

[13] See Section 3.

different rôle from the letters B, ..., K: it identifies the particular syntactical construction. Therefore, we get a sequence of four letters in each cell. T may occur in any of the four positions. Of the remaining letters none may be repeated on the same side of the T. Thus CTBD is permitted but CTBB is not.

All of this amounts to forming conjunctions, for example: CTBD which may be read: "C has B and D", can be interpreted as "Greatness has goodness and duration". The general form of this kind of conjunction is "x has y and z".

Llull has therefore not only gone beyond *predicates* involving only one argument, such as Man(x): x is a man, to *relations* between two elements, such as x is less than y, which are called *binary* relations, but even to *ternary* relations, such as $x + y = z$ or "John, Mary and Peter constitute a family". So we can say that here Llull has provided a rule, which we could describe as generating formulae involving binary and ternary relations (see Section 6 below).

There is one special restriction. Expressions such as "C has B and B" are not permitted but "C has D and C" is. This latter is of the form: "x has y and x". Likewise those of the form: "x has x and y" are also permitted.

Further, there is one major difference from modern usage. For Llull, two occurrences of x may be interpreted in different ways, i.e. chosen differently from the nine possibilities, whereas in modern usage any substitution of something for x requires that the same object be substituted at all occurrences. In addition to this we also may find the interpretation in an adjectival form in one place and in a noun form in another. Thus "C has D and C" has an interpretation "Greatness has duration and gluttony". However it would be misleading to write x_1 has y and x_2 which suggests (to the modern reader) that x_1 and x_2 may be different: a syntactic difference. For Llull the underlying formula is the same, it is just that the same letter is interpreted in two different ways: a semantic difference.

When T is the first or last letter, the remaining three letters must be distinct. One rôle of T is to solve a problem familiar to mathematicians. When one sees an expression $2 + 3 \times 4 + 5$, how does one evaluate it? The mathematician's approach is to write the expression unambiguously by using brackets. Thus we have variants such as $(2 + 3) \times (4 + 5)$ and $2 + (3 \times 4) + 5$, with the latter being unambiguous because $2 + (12 + 5) = (2 + 12) + 5$ by the associative law for addition: the order of the additions does not affect the answer.

In Llull's case when there are three letters to be combined, there may be zero, one or two letters to the left of the T and the other letters at the other side. His use of the letter T indicates the "bracketing". Thus BCTB could be written (BC)B in modern notation, whereas BTCB would be written B(CB). The combination TBCB is not permitted, but the combination TBCD puts all of B, C and D on a par. We could regard this as being B & C & D with our usual assumption that the associative law holds for conjunction.[14] That is to say, it does not matter which & we use first.

This "bracketing" makes the formulae read unambiguously. However, the interpretations are many since the letters on the left of the T are interpreted

[14] Informally, the grouping of the letters is irrelevant.

according to the first figure, and those to the right of the T according to the second.

Llull presents a table of 1680 combinations for his Fourth Figure, because the combinations are not simply of three letters but are mediated by his restriction that the same letter should not occur twice on one side of the "T" and also because of the way that one can interpret the same letter in different ways.[15]

Having produced this large number of questions and statements, in his *Ars generalis ultima*, [23], Llull seems to think that he has covered all eventualities. He says of the table for the Fourth Figure:

> *This table is a subject in which the intellect achieves universality ...*

To the modern reader this may be puzzling, since it is clear that the number of questions one might ask is without limit. However, if we look at his way of proceeding, then we see that for each definite finite number,[16] 2, 3, 4, he generates all the possibilities, that is to say, the pairs, triples, quadruples of letters.[17] The number of possibilities for each such definite number is finite because Llull has only a finite alphabet. This continues to be the case for any *specific* number of letters in a combination. It is only when one thinks of taking combinations involving arbitrarily large numbers of letters that there is the possibility of an infinite number of combinations–and there is no evidence that Llull ever had any reason to do that, nor that he even thought of doing so. He could already produce enough questions for his purpose of conversion to Christianity by using the combinations obtained from a relatively small number of letters.

6 Relations

It was Bonner who, in [3], first pointed out that Llull was using binary and ternary relations. He pointed his readers to the historical notes in [27], p. 299–300 where, incidentally, Llull is not mentioned. Llull appears to be the first person to introduce binary and ternary *relations* as opposed to *predicates*. Before that time, and certainly in Aristotle, predicates were only unary. Llull does not allow arbitrary binary and tertiary relations. He severely restricts them, but he does give schemata, which therefore means that there are several different relations. Thus in the third figure (see above Section 4), he has two types of binary relation. These differ only in that the order of the arguments is reversed. In Llull's notation this is the difference between the orders BC and CB. In the fourth figure there are ternary relations. These differ in the order of the letters and they are determined by the location of the letter T. Thus BTCB is very different from BCTB as we have noted above in Section 4. Even more different are BTCD and BCTD where we genuinely have three different letters,

[15] The best explanation of the number 1680 that I have found is in Eco [11], pp.61–2.

[16] For a more precise discussion of the phrase "definite finite number" see my [9], Chapter I.

[17] Leibniz used the neologisms com3nation, com4nation, etc. for these combinations that involve only a specific number of letters in his [20].

but note our comment above in Section 5, about the different interpretations of two occurrences of a letter such as B in a Llullian formula.

This move frees formal logic from the tyranny of relations (or should I say, predicates) having only one argument. Incidentally, once one has two or more arguments then it is possible to build more complicated relations, for example (in modern notation), $P(x, y)$ & $Q(y, z)$ builds a ternary relation between x, y and z, from two binary ones. Such constructions are impossible with predicates of one argument only. Llull does not go so far, so his sentences correspond to the forms $P(x, y)$ or $R(x, y, z)$ (with P and R being constant relations) and more complicated arrangements of relations are not considered.

7 Substitution

There has been significant discussion in the literature regarding the earliest date for the introduction of variables into mathematics.[18] Although Euclid used letters for quantities and points, each of these seems to have indicated a definite, though arbitrary element. Some authors have argued that variables were used by Diophantos, early in the Christian era (perhaps *c.* 300AD). He was interested in the solution of an equation (as we would express it today). Therefore there was an unknown element. He used a symbol, which seems to be distinct from the letters of the alphabet, for such an unknown. This may have been an abbreviation for a word meaning, in essence, "unknown". see [9], p.70. However, the symbol did not have a variety of interpretations: it simply stood for a quantity that was required to be determined. Other symbols were used for various quantities. Diophantos used $\Box$ for the square of the unknown, i.e. the number multiplied by itself. This tradition continued through al-Khwarizmi [30] of the ninth century and into the Italian algebraists of the sixteenth century, where the unknown was called the *cos* or "thing". There was no suggestion that the value of an expression, say one equivalent to $x^2 + 10x$ might change, rather the emphasis was on solving problems expressible by equations such as $x^2 + 10x = 39$.[19] It is the sixteenth century Italian, Maurolico (1494–1575) [26], who is generally credited with the introduction of variables as mathematicians now use them. The full use of variables in algebra was not to emerge until the late sixteenth century in the work of Viète [36]. See especially his *Æquationum recognitione et emendatione*, [35] and [9] p.99, [7], pp.14–15. In his *In artem analyticam isagoge* Viète did not exploit variables very much but he did use them to denote differing quantities as opposed to Cardano, [6] (also sixteenth century), who only used letters to denote specific unknowns or powers (squares, cubes) of the unknown. For example on p.190 of [35], Viète is using variables as such.

[18] The concept of variable is still difficult for school students to grasp, which may indicate that it is much more complicated than mathematicians usually assume.

[19] This, incidentally has the solution $x = 3$ but it also has a negative solution, which was not accepted for a long time, $x = -13$. See [30] and [9], p.68.

Variables were also used in literature to symbolize specific meanings. For example the “Tower of Wisdom” (*Turris Sapientiae*) of c. 1300 (see [33]), has twelve letters each of which has ten interpretations.

In logic the use of letters as variables is hard to pin down, since the development was a very slow one. We notice it most in work of Leibniz [19] in the seventeenth century.[20] Leibniz’s work in logic remained unknown for centuries. In the nineteenth century we find Boole [5], developing what was to become the propositional calculus.[21]

However we find something rather different from the modern view in Llull’s use of letters. He uses the letters B, C, D, E, F, G, H, J, K to stand for different things, but even in the same statement they may take on different values at different times. This should be opposed to the situation in modern mathematics and logic where, for example, when we substitute for x in, say, $x^2 + 10x$ we replace each occurrence of x by the same number. For Llull, each letter has *several different interpretations*. There are therefore two differences even from the earlier uses in mathematics. First, the symbol was denoting *the* object, or number, to be found, and secondly, the symbol always represented this same number throughout the statement or argument.

Llull has not explicitly said so, but I would say that Llull was *substituting* words for these letters. Here I diverge from Bonner [3], who says “the letters don’t represent variables but constants”. Perhaps the best way to describe the situation is to say that there are several *different* constants corresponding to each letter. Moreover Llull has not just one possibility but a variety of possible substitutions: six in each case.

8 Machines

The idea of movement is very important in distinguishing between Llull’s first figures and his fourth one. The involvement of movement in calculation is immensely ancient: for example, scratches on bone from Palaeolithic times (see [25]) involve movement. In historic times the system[22] found in the work of the Venerable Bede (c. 673–735), allowed representations using various configurations of the hand for (individual) numbers up to 1 000 000. Symbolism combined with, indeed directing, movement is found from the tenth century in the correlation by Guido d’Arrezzo of points on the hand with musical intervals, and moving the finger of the other hand to point to these various locations. It is therefore

[20] Aristotle’s use of letters seems to be of a different nature. In their cases each letter represents a specific, but perhaps unknown, element. In the case of Llull, Leibniz, etc. the same symbol (letter) takes on different values at different times.

[21] I have found no other acknowledgment of this work of Leibniz before 1900. Boole worked in the nineteenth century, and although Boole had access to some of Leibniz’s work, it seems that it did not have any influence upon him. See Grattan-Guinness [14], p. xliii and also the *Encyclopedia Britannica* article by Dipert [10].

[22] See, e.g., Science and Society Picture Library 2004 at `http://www.scienceandsociety.co.uk/results.asp?image=10308471&wwwflag=2&imagepos=1`

surprising that Yates [39], p. 176 says: “Finally, and this is probably the most significant aspect of Lullism in the history of thought, Lull introduces movement into memory.” What does seem to be the case is that Llull introduces a machine with moving parts into logic, and thereby, memory. It therefore seems both useful and indeed necessary to distinguish a “machine” as comprising a physical object in which there are parts that are movable with respect to each other, and this should be opposed to movement, in which the user moves separate individual objects, or his or her configuration relative to an object (as with the Guidonian hand).

Llull’s first two figures had no moving parts. However the fourth figure had two discs independently revolving around a central axis. It therefore seems appropriate to call this a “machine”. By rotating the discs, different alignments between letters (or words) on them yielded the differing combinations that we have referred to above.

Thus the machines produced results. These results then had to be interpreted (by substituting for various letters) and these then gave statements (or questions). Such statements were sometimes true and sometimes false. However the machines did not determine which. It can therefore be said that the machines aided the logic (and the logician) but they did not do all the work, they did not perform all the calculations. I would therefore say that Llull did not produce a calculus. What he did produce was a system and machine for generating sentences: statements or questions.

It is perhaps worth mentioning that the question of input and output from any machine is a separate question from the workings of the machine. This is as true for today’s computers as it is for Llull’s machines. For a modern machine there has to be an input provided. This is usually done, or at least, stimulated, by punching in various characters on a keyboard. The keyboard is not part of the computation. Likewise, when we need an output this is achieved, normally, either by getting an image on a screen or by printing something out on paper. Again these are not part of the computation process. (Although, when one purchases a computer, it usually comes with a keyboard and a screen, these two components are actually peripheral to the actual computer and are indeed referred to as “peripherals”, just as a printer is.)

I would therefore argue that it is accurate, and in accordance with modern usage, to say that Llull “computed results”. But I would also stress that the results that he computed were not the final word. In addition to his computation process he had to provide (or wished the users of his system to provide) the necessary arguments to determine the truth or otherwise of the statements generated from his computations.

The idea of Llull of making more and more complicated combinations was taken up, as is well known, first by Kircher [16, 21] and then by Leibniz in his *Ars Combinatoria* [20].

Indeed Leibniz devotes considerable space to Llull. On pp. 39–40 of [20] he is critical, as we were above, of Llull’s restriction to nine letters. He asks in particular why Figure and Number are not included among Llull’s basic terms.

Then Leibniz proceeds to investigate combinations as such, developing his calculations to larger sets of basic elements and imposing conditions on repetitions. This was purely mathematical work. It was only later that Leibniz developed the idea of the *Ars combinatoria* in the sense of being able to resolve logical disputes by taking up pens and calculating. (The famous *Calculemus* quotation is the culmination of this as an idea, see [18].)

On pp. 51–52 of Gerhardt's edition of the *Ars combinatoria*, [20], Leibniz discusses the "wheels" (*rotae*) of Llull and his successors. Later Leibniz was himself to design a machine for computing, in this case, for the four basic arithmetic operations $+, -, \times, \div$ using wheels. There seems to be a much stronger case for calling this a "computer" than there is for so labelling Llull's machine.

We also note that Leibniz included a *Demonstratio Existentiae Dei* in his work *Ars Combinatoria*. This seems to have been added as a separate exercise. It is listed at the very end of Leibniz's list of contents yet occurs at the beginning of the work. This "proof" does not use combinations at all, but it does use the kind of logical argument that Llull uses to search out the truth of statements produced by his machines. Such an argument goes from axioms in a logical way, quoting which axioms, or previously deduced statements, are being used at each stage of the proof. This clearly foreshadows the rules of formal logic and the style of reasoning that we use today, which were introduced by Boole, Frege and Russell [5, 12, 37]. Leibniz writes on formal logic with a symbolism that is close to today's in his manuscripts [19] and [18]. These works are self-contained and do not in fact refer to Llull, but it is hard to imagine that Leibniz's beginnings of logic in terms of a formal language were not also influenced by Llull.

9 Computer Science and Computer Engineering

Definitions of new disciplines take some time to stabilize and it is certainly not clear that either of the terms "Computer Science" and "Computer Engineering" has so far stabilized. "Computer Science" may be thought of as the science of computing, including the study of the behaviour of computing machines and their capabilities. "Computer Engineering" centres on the construction of computing machines. Both are inextricably involved in the design and construction of programming languages, the formal languages that we use with computers, though this tends to be more the concern of Computer Science.

Claims have been made that Llull was the first computer scientist, for example, by Sales [32]. It was Martin Gardner [13] who was the first modern to draw attention to Llull in the context of computing, and he wisely only refers to Llull making "logic machines", not "computers".

Llull certainly made machines. He was also involved with the design of what is now described as a "formal language". The machines did actually compute results. The fact that they were relatively simple results: the combinations of two or three letters (from an alphabet of, in the last analysis, nine), should not detract from Llull's inventiveness.

We agree with Colomer [8], p.132, who writes: *Hi ha primerament una coincidència de caràcter extern, però de conseqüències incalculables cara al futur: la formalització del llenguatge, o sigui, la creació d'un llenguatge artificial, en el qual els signes substituteixen les operacions del llenguatge comú.* We therefore conclude that Llull could be described as a "computer engineer" in that he made logic machines which, after all, was what Turing [38] did last century. But Llull made significant contributions to what is now known as "computer science" in that he developed the idea of formal language (formal combinations of symbols), contributed to the use of variables and introduced, albeit to a limited extent, binary and ternary relations.

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